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THE FORMULATION OF EXPECTATIONS
IN DYNAMIC SYSTEMS

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I. Introduction and Summary

Since Keynes' General Theory, economists have emphasized the importance of expectations to the dynamics of macro-economic behavior. The economics approach to treating such phenomena, however, often fails to enhance our understanding of dynamic systems. This paper contrasts the system dynamics modeling philosophy with models of expectations found in the economics literature. The focus is on the formulation of expectations because of its dynamic importance, but the methodological analysis applies over a wider range of issues. This work is part of an on-going project in the System Dynamics Group at M.I.T. to compare and contrast various approaches to social system modeling.

Part II describes certain principles that are essential to system dynamics models. They are:

(1) System dynamics adopts a systems point of view.

(2) Social systems are state-determined and, as such, are dynamic, continuous, and structured by a simple organizing concept.

(3) Models should emphasize causal structure.

(4) Models should retain the notion of conserved flows under certain conditions.

(5) Model equations should represent actual processes as they are observed by participants in the system.

(6) System dynamics models are policy-oriented.

(7) Non-linearities are important to system modeling and model behavior.

In Parts III and IV, these principles are applied to several different models of expectations. Part III describes Hicks' model, as set forth in Value and Capital. Hicks used comparative statics, which reveals assumed equilibrium conditions but obscures the dynamic processes. A later analysis of Hicksian expectations by Arrow and Nerlove introduces a dynamic component but retains an important static element. They use their model to justify "adaptive expectations", which, as they show, maintains stability in their model under all conditions. They also claim that the adaptive formulation is a "reasonable" reflection of reality, because of the implied assumption that stability characterizes the real economic system.

Part IV takes up an analysis of the Arrow and Nerlove system. Section A models a single-product version in a system dynamics format and shows that the model is not consistant with the principles of modeling established in Part II. Section B extends the original structure in order to incorporate some of these principles. The extended model is described in detail and then simulated in order to compare the effects of various alternative formulations of the expectations process. The simulations show that the modified system with adaptive expectations is much less stable than it is with some of the other formulations.

Section C examines two more model extensions. Their purpose is to show that small changes in structure, in accordance with the established system dynamics modeling principles, have major implications about alternative expectation formulations. The first change results in

dynamic behavior modes that differ among the various types of expectations and are very different from those exhibited in Section B. With the second change, overall system behavior is virtually the same for all of the widely different expectation models.

The analysis of macro-economic systems in terms of dynamic, causal structures differs from the approach taken in most economic models. Part IV carries out such an analysis in order to reveal these different modeling viewpoints. In the process, we discover that the way expectations are formed may have a stabilizing or de-stabilizing effect on economic behavior, depending on the structure in which they are embedded. Conclusions about the stability effects of adaptive or other types of expectations cannot be generalized to all economic systems, but must be made only with reference to particular closed-loop models.

II. System Dynamics Principles of Social System Modeling

The model-builder often approaches his task with a set of modeling principles, or biases, which color his world-view, problem definition, and even the model conclusions. These "priors", as Urban[1] calls them, should be recognized openly in order to facilitate understanding of the resulting models.

The system dynamics priors are discussed below. They do not constitute a complete list of criteria necessary for good social science models, but are consistent with such lists.[2] The priors considered here are essential to system dynamics and are violated by the great majority of economic models, including those that deal explicitly with expectations. In the discussion that follows,[3] no attempt is made to "justify" the biases or to "prove" that they are correct. To some extent, one can do little more than assert that real social systems belong to one class of systems as opposed to others, and the reader must choose.

(1) System dynamics adopts a systems point of view, which focuses on all critical aspects of a problem whether or not they belong to a particular academic discipline or are objectively measurable.

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[1] Urban, Glen L. "An Emerging Process of Building Models for Management Decision Makers", A.P. Sloan School Working Paper 591-72 (Cambridge: M.I.T., 1972).

[2] See, for example, Little, J.D.C. "Models and Managers: The Concept of a Decision Calculus", Management Science, Vol. 16, No. 8, 1970.

[3] In his unpublished Ph.D. Dissertation, Jørgen Randers has a similar list of system dynamics priors, although his list is organized somewhat differently and does not include points 4-6. See Randers, J. Conceptualizing Dynamic Models of Social Systems: Lessons from a Study of Social Change. (Cambridge: A.P. Sloan School of Management, M.I.T., 1973), pp. 44-5.

The system "prior" requires a precise definition of system boundary and clear identification of which variables are endogenous (affecting and affected by other system variables) and which are exogenous (affecting but not affected by the system variables).

(2) System dynamics asserts that social systems are <u>state-determined</u>. Real-life change occurs through processes of integration which can be captured mathematically by a set of differential equations:

$$\frac{d}{dt}\underline{X} = f(\underline{X},\underline{Z},\underline{U}),$$

where $\underline{X}$ is a vector of state variables, $\underline{Z}$ consists of exogenous inputs, $\underline{U}$ is a noise vector and f (·) is a vector-valued function determining the instantaneous flow rates as functions of the states. Formal models in the physical sciences normally consist of differential equations; but in economic modeling, simultaneous equation systems are more common:

$$\underline{X}_t = f(\underline{X}_t, \underline{Z}_t, \underline{U}_t),$$

where $\underline{X}_t$, $\underline{Z}_t$, and $\underline{U}_t$ are vectors of endogenous, exogenous (including lagged endogenous) and stochastic variables respectively. Systems containing simultaneity, i.e. where the functional argument $\underline{X}_t$ is a non-zero vector, do not necessarily contain integration processes. Senge[4] shows, in fact, that there is no necessary correspondence between models belonging to this class and the underlying "integration-feedback" structures existing, according to the system dynamics prior, in real life.

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[4]Senge, P. "Econometric Least Squares Estimation of Dynamic Systems: An Appraisal", System Dynamics Group Memo, D-1944-1; see especially Part II "The Theory of Integration-Feedback Systems".

The characteristics of differential equation representation give rise to other priors which can be listed separately or, as is done here, can be considered as part of the state-determined principle. They are that real systems, and therefore formal models, are dynamic, continuous and structured by a very simple organizing concept. The dynamic, or time-varying, approach is captured in the time-derivative ($\frac{dX}{dt}$) and emphasizes system behavior over time rather than static properties existing at any point in time. The continuous representation reflects the claim that real processes are more nearly continuous in their evolution than discrete. Even though discrete events are observed, the processes of information perception, manipulation and evaluation leading up to particular events is far more continuous than the occurance of a culminating event might suggest.[5]

Finally, the organizing concept implicit in state-determined systems is the division of all variables into two categories: rates of change ($\frac{dX}{dt}$), which are system actions dependent on conditions at each point in time; and levels, or states ($\underline{X}$), which indicate the system condition over time. The rate-level dichotomy is discussed more fully in Part IV.

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[5]Continuous representation is not a necessary feature of state-determined systems, which can be modeled discretely. What is essential to the state-determined model is the process of accumulation. Retaining the notion of continuous processes, however, helps to avoid the pitfalls of many discrete econometric models, whose periods are determined in accordance with the sampling intervals of available data, rather than with respect to the underlying integration-feedback processes generating the data.

(3) Great emphasis is placed on developing a causal structure, a network of causal relationships embedded in feedback loops that determine system behavior.[6] Structure is fundamental because it represents explicitly relatively invariant causal connections perceived by the modeler to be significant. Causal structure implies dynamics, in that to be causally determined a change in one variable must follow in time the change in the causal variable. This principle is violated in economic estimation models which confuse correlation with causality, and in pedagogical models which confuse independent and dependent variables. The usual static demand and supply schedules, for example, cannot indicate whether price is causally dependent on quantity, or vice-versa.

(4) A basic principle in the physical sciences is the conservation of matter. In system dynamics modeling the principle of conserved flows is retained when a physical process is considered endogenous to the system being modeled.[7] This means that flows such as the production of goods or the placement of orders are accumulated in levels, e.g. goods inventories or order backlogs, and that these levels are depleted by outgoing flows which, in turn, are limited by the content of the level.

(5) Model equations should represent actual processes as they are observed by participants in the system. In the same vein, Little writes that "...coefficients and constants without clear operational interpretation are to be discouraged."[8] Economic models, in contrast, often imply a set of operational,

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[6] Feedback loops are described in detail in Forrester's books, especially in Principles of Systems, Chapter 2; Part IV of this paper gives examples.

[7] An interesting violation of this principle is found in mathematical models of the famuliar multiplier-accelerator interaction. See Low, G.W. and Mass, N.J. "A Systems Approach to the Multiplier-Accelerator Theory of Business Cycles", System Dynamics Group Memo D-1785-1

[8] Little, op. cit., p. 5 (of the article).

or behavioral, postulates which enable the modeler to assume simplifying conditions and estimate parameters and functions which are meaningless as they stand. For example, the estimation of simultaneous equation systems is usually based on equilibrium conditions, behind which lie crucial assumptions about competition, utility maximization, etc. The model is determined by the assumed behavior, but does not represent it explicitly.

An example that is particularly relevant to this paper is the concept of "rational expectations," introduced by Muth in 1961.[9] Broadly speaking, Muth claims that "the market expectation of any relevant variable must represent the best forecast that could be made of that variable, on the basis of all the information available at the time of the forecast."[10] This requires that

$$_{t+1}P^*_t = \underset{t}{E}[P_{t+1}],$$

where $_{t+1}P^*_t$ is the price expected for time t+1 at time t, and $\underset{t}{E}[P_{t+1}]$ is the conditional expectation of P_{t+1} formed using all information about the exogenous and endogenous variables available as of time t.

Expectations are considered "rational" in the sense that, on average, people use all available information efficiently; that expectations are informed predictions of the future and are essentially the same as the prediction of the relevant economic theory; and that if the prediction of the theory were substantially better than the expectations of the firms, profit

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[9] Muth, J.F. "Rational Expectations and the Theory of Price Movements," Econometrica, Vol. 29, No. 3, July 1961, pp. 315-335.
See also Modigliani, F. and Shiller, R.J. "Inflation, Rational Expectations and the Term Structure of Interest Rates", Economica, Feb. 1973, pp. 12-43; Sargent, T.J. "Rational Expectations, the Real Rate of Interest, and the 'Natural Rate' of Unemployment", Brookings Papers on Economic Activity, No. 2, 197

[10] Muth, op. cit., p. 28.

opportunities would attract "insiders" and diminish the gap. This model of expectations is derived from the model (the theory) of the relevant system, so that the function describing expected price, for example, does not directly represent the way price expectations are formed by people in the system. Muth writes that the rational expectations model "does not assert that the scratch work of entrepreneurs resembles the system of equations in any way...."[11] This approach differs from system dynamics models, which usually include explicitly the behavior or decision mechanisms considered important to the process or problem being modeled.

(6) One uses system dynamics to build policy models. They are models of particular problems and as such seek policies, or decision rules, to eliminate undesirable modes of behavior. The analysis focuses on characteristics of system behavior, including, for example, the amplitude or periodicity of oscillations, the delayed reaction of one system element to changes in another variable, damping ratios and phase shifts. A policy model is concerned with indicative system behavior and not with making particular point predictions or fitting data according to statistical measures of fit. The modeling prior described here is not explored in this paper, although the policy orientation of system dynamics models could not be overlooked when applying the various expectation formulations to specific situations.

(7) Non-linearities are important to understanding system behavior. In representing causal relationships in a dynamic model, one cannot assume that the system remains, under all possible conditions, within linear ranges. For

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[11] Ibid., p. 317.

in doing so, one is building into his model artificial constraints on system behavior, rather than allowing behavior to be determined by the perceived causal relationships. Most economic models assume that economic systems remain close to equilibrium, with deviations representing only stochastic inputs. Such models assume linearity in the parameters for analytical convenience and justify linearity by the system's supposed proximity to equilibrium. This supposition, in turn, is justified by an appeal to established theory rather than by direct observation.[12] Once a system dynamics model is constructed, the resulting behavior, under various test cases, might reflect excursions of the system over only the linear or near-linear ranges. One does not assume such behavior, however, when constructing the model components.

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[12] For a detailed critique of such reasoning, see Morgenstern, O. "Thirteen Critical Points in Contemporary Economic Theory and An Interpretation", Journal of Economic Literature, Vol. X, No. 4, Dec. 1972, pp. 1163-1189.

III. Hicks' Model of Expectations

Expectations are central to the macro-economic theories of Keynes and Hicks. Before their time, expectations received little attention, except in Irving Fisher's writings on interest rates. In *Value and Capital*, Hicks combines verbal treatment, used exclusively in Keynes' *General Theory*, with formal mathematical analysis of expectations and dynamic processes. His use of comparative statics tends to diminish the richness of the verbal description and would be of historical interest only, were it not for the important role of static analysis in neo-classical growth models and general equilibrium statistical models.

In *Value and Capital*, Hicks confined dynamic processes, such as the formation and influence of expectations, to a comparative statics framework. A later analysis of Arrow and Nerlove rescued the expectations formation process from static analysis but by no means relinquished the use of comparative statics. This part of the paper describes the Hicksian model of expectations and Arrow and Nerlove's modified version.

A. A Comparative Statics Treatment of Expectations

Hansen describes comparative statics as follows:

> In static analysis, certain parameters such as tastes, income, etc., being assumed as given, a functional relation is posited between two variables, say price and quantity demanded. At a higher price less will be demanded. But this is purely static analysis. If a change in anticipations is introduced so that prices are expected to rise further, Demand will probably increase -- *more* will be purchased in anticipation of further price increases. This represents a dynamic situation. The *change* from one equilibrium position to another is the subject matter of comparative statics. Comparative statics is a study of "the way in which our equilibrium quantities will change as a result of changes in the parameters taken as 'independent data'" ....Comparative statics leaps over the time involved in the transition to the successive positions of equilibrium .. . [It] "involves the special case where a 'permanent' change is made, and only the effects upon final levels of stationary equilibrium are in question."[13]

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[13] Hansen, A.H. *A Guide to Keynes* (New York: McGraw Hill, 1953), pp. 45-46. The passages in quotes are from Sa[illegible]

In the conventional market economy models, prices adjust rapidly relative to tastes, incomes, productive capacity and other determinants of demand and supply; so that all changes in price and quantity can be assumed to occur along (temporarily) static demand and supply curves. To the conventional model, Hicks adds "anticipations", which are formed concurrently with prices. He writes, "I assumed the process of adjustment to temporary equilibrium to be completed within a short period (a 'week'), while I neglected the movement of prices within the week, so that my economic system could be thought of as taking up a series of temporary equilibria".[14]

Each "Monday" of this arbitrary "week", all markets are cleared, on the basis of previously-determined levels of capital and resource allotments. Between these clearings, which are treated mathematically as if they occur instantaneously, expectations are formed and plans are made. The resulting plans determine the new market prices established at the beginning of the succeeding week. Thus,

> in determining the system of prices established on the first Monday, we shall also have determined with it the system of plans which will govern the distribution of resources during the following week. If we suppose these plans to be carried out, then they determine the quantity of resources which will be left over at the end of the week, to serve as the basis for the decisions which have to be taken on the second Monday. On that second Monday a new system of prices has to be set up, which may differ more or less from the system of prices which was established on the first.[15]

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[14]Hicks, J.R. Value and Capital (Oxford: Clarendon Press, 1939), p. 336.

[15]Ibid., pp. 131-2.

In this economic system all markets clear continuously and instantaneously while other processes such as capital accumulation work themselves out over the longer term. Expectations are assumed to be formed instantaneously, along with price and demand.

During any given "week", a change in price results in a total and immediate change in one's expected price or set of future prices. In this framework, Hicks defines the "elasticity" of expectations (with regard to expected prices) to mean "the ratio of the proportional rise in expected future prices of X to the proportional rise in its current price".[16]

Mathematically, the elasticity of price expectations can be expressed as

$$\epsilon = \frac{d(EP)}{dP} \cdot \frac{P}{EP}$$

Hicks says that the two pivotal elasticities are $\epsilon = 0$ and $\epsilon = 1$. When $\epsilon = 0$, expectations are rigidly inelastic with respect to price and, assuming no other influences, may be considered constant. When $\epsilon = 1$, a change in current prices will change expected prices in the same direction and in the same proportion; i.e., if prices were previously expected to be constant at the old level, they are expected to be constant at the new level. Hicks mentions three other ranges of elasticities, the two extremes of $\epsilon > 1$ and $\epsilon < 0$, and the intermediate case of $0 < \epsilon < 1$. When $\epsilon > 1$, people recognize a trend and try to extrapolate; when $\epsilon < 0$, "they make the opposite kind of guess, interpreting the change as the culminating point of a fluctuation".[17]

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[16] Ibid., p. 205.

[17] Hicks, op. cit., p. 205.

Except for $\epsilon = 0$ and $\epsilon = 1$, the Hicksian elasticities seem to imply that the past history of prices is necessary in the formation of current expectations, especially, for example, when people are looking for trends or turning points, but also in the case of $0<\epsilon<1$, where expected price changes are less than proportional to price changes. The comparative statics method, however, does not permit such memory or accumulation of historical information. For, if current prices were to influence expectations established in the future, then the "once-for'all" price changes at each instant in time would no longer yield concurrent equilibria in expected prices or in other variables affected by expected prices.

On the other hand, Hicks' verbal treatment of expectations clearly attributes great significance to past prices. He writes, for example:

> We must never forget that our 'week' is arbitrary in length; this is of great importance in the formation of expectations. The elasticity of expectations depends upon the relative weight which is given to experience of the past and experience of the present; now if the 'present' is taken to cover a longer period of time, 'present experience' will necessarily weigh more heavily, and (even in the same psychological condition) expectations will tend to become more elastic.[18]

B. A Restatement of Hicksian Expectations

By focusing on Hicks' descriptive, rather than analytical, treatment of expectations, Arrow and Nerlove[19] transform Hicksian expectations into an explicitly dynamic framework. They claim that Hicks' description is best represented by the "adaptive expectations" formulation, which expresses expected price as a weighted average of past prices, with the weights declining geometri-

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[18] *Ibid.*, p. 272.

[19] Arrow, K.J. and Nerlove, M., "A Note on Expectations and Stability," *Econometrica*, (Vol. 26, No. 2, April 1958), pp. 297-305.

cally as one goes back in time.[20]

The authors show that Hicks' definition of elasticity can be expressed in difference and differential equation form, if we accept the concept of a single expected price based on weighted past prices. In difference equation form, elasticity (ϵ) is now defined differently from Hicks' static definition:

$$\frac{EP(t) - EP(t-1)}{P(t-1) - EP(t-1)} = \epsilon \tag{1}$$

where EP(t) is the normal expected price of a commodity during period t, P(t) is the actual price, and prices are expressed in logarithms. When expressed as

$$EP(t) - EP(t-1) = \epsilon[P(t-1) - EP(t-1)], \tag{2}$$

it can be seen that (1) defines "adaptive expectations;" expectations are changed (or adapted) in each period by a constant fraction of the difference between actual and expected prices of the previous period.

In difference equation form, the two pivotal elasticities of $\epsilon=0$ and $\epsilon=1$ have the same interpretations as those contained in Hicks. If we examine the differential equation analogue of (1), however, the case of $\epsilon=0$ has the same meaning but $\epsilon=1$ does not. Dropping the logarithmic representation in (1) and observing the i^{th} commodity, we have,

$$\dot{EP}_i = \epsilon(P_i - EP_i) \ , \ \epsilon \geq 0. \tag{3}$$

In the static case, where $EP_i = P_i$, Hicks' elasticity was unity, whereas

[20]The notion of one expected price rather than a set of expected prices can be justified, according to the authors, if we consider not the expectations of particular future prices, but "expectations of the average level about which future prices are expected to fluctuate," what they call "expected normal price." (Ibid., p. 298).

in the differential equation $\dot{EP} = P$ when $\epsilon = +\infty$, as can be seen from the solution for EP:

$$EP(t) = EP(0)\ e^{-\epsilon t} + e^{-\epsilon t} \int_0^t \epsilon P(u) e^{\epsilon u} du . \qquad (4)$$

In this case, the time constant, which is the inverse of ϵ, approaches zero as $\epsilon \to \infty$. If the initial value of EP(=EP(0)) is in the distant past, the first term in equation (4) is negligible; and EP(t) may be taken as an exponentially weighted average of past prices.[21]

Having established that all non-negative expectation elasticities, as defined by Arrow and Nerlove, can be expressed in the adaptive expectations format, the authors examine the dynamic effects of expectations on a particular multi-product model. The model has two dynamic equations. One defines the change in expected prices, according to the adaptive expectations format of equation (3). The other defines the change in prices:

$$\dot{P}_i = K_i X_i \qquad (5)$$

where X_i is the excess demand for the i^{th} commodity, and K_i is constant. The determination of excess demand is instantaneous rather than dynamic:

$$X_i = X_i(P_i, \ldots, P_n; EP_1, \ldots, EP_n). \qquad (6)$$

At equilibrium $X_i = 0$ and $EP_i = P_i$; and expanding (6) around the equilibrium price (P^o) gives the linear approximation:

$$X_i = \sum_j a_{ij}\ (P_j - P_j^o) + \sum_j b_{ij}\ (EP_j - P_j^o),\ (i = 1, \ldots, n) \qquad (7)$$

where $a_{ij} = \partial X_i / \partial P_j \begin{cases} \geq o \text{ for all } i \neq j \\ < o \text{ for } i = j \end{cases}$

and $b_{ij} = \partial X_i / \partial EP_j \geq o$ for all i and j.

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[21] As $\lim_{\epsilon \to \infty} e^{-\epsilon t} \int_0^t \epsilon e^{\epsilon u} du = 1$

Equation (7) can be substituted into (5) to give $\frac{dP}{dt}$ as a function of price and expected price.

The authors show mathematically that "under adaptive expectations, a [particular] dynamic system, stable under static expectation, remains stable no matter what the inertia of the system or the elasticities of expectations."[22]

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[22] Ibid., p. 304.

IV. A System Dynamics Treatment of Expectations

A. The Single Product Price-Demand Model

Hicks used comparative statics to portray a sequence of "snapshots" of market conditions over time. Arrow and Nerlove combine comparative statics with explicit dynamics by portraying "excess demand" ($\underline{X}_i$) as an instantaneous function of dynamically-determined price and expected price.[23] At each point in time, static excess demand schedules are described by the slopes a_{ij} and b_{ij} given in equation (7). The static representation of the market response to price changes, used within the framework of a dynamic model, has important dynamic implications. Samuelson writes, "We find ourselves confronted with this paradox: in order for the comparative statics analysis to yield fruitful results, we must first develop a theory of dynamics."[24]

In the present case, the substitution of an equilibrium model for the dynamics of supply and demand might be justified if the subsystems determining the X_i are assumed to respond and equilibrate very rapidly compared to the rest of the system. In the Arrow and Nerlove model, excess demand, which may be defined as the net of purchases and production, is reduced because prices and expected prices change rapidly compared with other determinants of supply and demand that are left out of the model (such as income or capacity acquisition). The static representation of excess demand assumes

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[23] More recent studies in macro-economic growth theory also combine time-varying processes with instantaneous market clearings. See for example, Foley, D. and Sidrauski, M. Monetary and Fiscal Policy in a Growing Economy. (New York: MacMillan, 1971), in which differential equations define the growth of capital and government debt, while asset markets clear instantaneously, using capital and debt as given data at each moment in time.

[24] Samuelson, P.A. op. cit., p. 262-3.

that variables other than price or expected price have no significant influence, that they can exert their impact only exogenously, or that they change very slowly relative to the time required for the explicitly represented processes to equilibrate. "*If one can be sure that the system is stable and strongly damped*, there is no great harm in neglecting to analyze the exact path from one equilibrium to another ...."[25]

But if important interactions occur between non-price determinants of supply and demand on the one hand and prices and expected prices on the other, during the time it takes actual and expected prices to adjust to new conditions, then a comparative statics treatment of excess demand can be misleading. Suppose, for example, that the response time in forming expectations adaptively is relatively long (ie., that ϵ is closer to zero than to one); and that during the expectations adjustment period, changing prices encourage producers to hold back inventories from the market or to add quickly to productive capacity, e.g. by hiring more labor. Then the model of excess demand contained in equations (6) and (7) would be misleading. The Arrow and Nerlove equations imply that such interactions do not occur.

By representing excess demand statically, and thus assuming rapid damping, Arrow and Nerlove are able to use algebraic arguments to prove that their dynamic system using adaptive expectations is stable. But their conclusion is determined in part by an *a priori* assumption about market stability. They imply, in fact, that their conclusion about the formation of expectations can

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[25]*Ibid.*, p. 331 (Italics added).

be generalized to the actual macro-economic system (which is taken to be strongly damped), when they write that adaptive expectations act as a stabilizer and are therefore more "reasonable" than other formulations.[26]

Consideration of the Arrow and Nerlove model in light of the "priors" established in Part II will be easier if we use only the simplest version of their system, the near-trivial case of a single-product price and market. Arrow and Nerlove's mathematical analysis is lost by using the simple version, but not our understanding of the modeling issues discussed above. The Arrow and Nerlove system contains dynamic equations and can be recast in a system dynamics format. The process of doing this should reveal how the model deviates from the system dynamics priors.

Part II asserts that social systems, like physical systems, are state-determined, continuous in their evolution, causal, and usually non-linear. A model that can capture these features contains four essential elements:[27]

(1) Levels, which are accumulations within the system;

(2) Flows that transport the contents of one level to another;

(3) Decision functions that control the rates of flow between levels;

(4) Information channels that connect levels to the decision functions.

To represent a single product version of the Arrow and Nerlove model in a system dynamics structure, one must indicate the causal relationships and

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[26] Arrow and Nerlove, op. cit., p. 304.

[27] The basic structure of system dynamics models is discussed in Forrester, J.W., Industrial Dynamics. (Cambridge: M.I.T. Press, 1961), pp. 67-72.

identify the levels, flows and information links. Causality can be represented in the diagram shown in Figure 1. Each arrow represents a direct causal linkage between two variables. The signs indicate that a change in one element causes a change in the other element in the same (+) or opposite (-) direction.

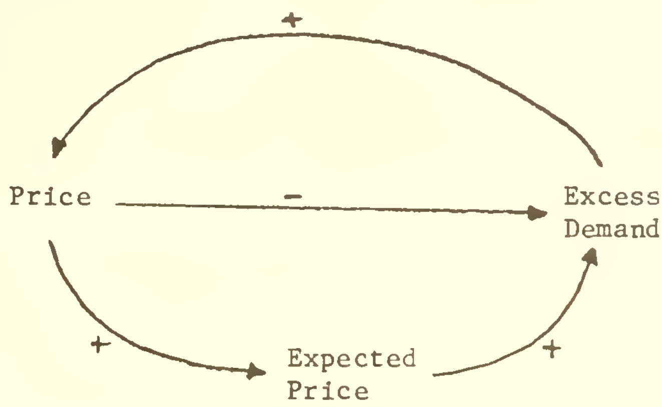


Figure 1 Causal Loop Diagram of a Simple Price-Demand Model

In this closed-loop structure, an exogenously-determined increase in price causes expected price to rise, because of the exponential smoothing indicated in equation (4). This, in turn, causes excess demand to increase (because in equation (7), $b_{ij} = \frac{\partial X_i}{\partial EP_j} \geqslant o$ for all i and j). At the same time the higher price imposes a negative influence on excess demand (as $a_{ij} = \frac{\partial X_i}{\partial P_i} < o$, for i = j). The resulting change in excess demand feeds back to price in the indicated manner.

A problem with this causal loop analysis is the simultaneous formulation of excess demand. Part II asserted that "causality implies dynamics", yet here there are no dynamics in the market equation. Price, in fact, responds dynamically to excess demand, but there is no dynamic causality flowing in the opposite direction. This becomes clearer if we

identify the [illegible] diagrams [illegible]. Presently [illegible] in the diagram shown in Figure [illegible] [illegible] [illegible]

[illegible]

identify the levels (stocks) and rates (flows) in the Arrow and Nerlove model. The differential equation defining the change in price ($\dot{P}$ in equation (5)) indicates that $\dot{P}$ (= dP/dt) is a rate of change, or flow, which depends on the price level.[28] In like manner, expected price, according to the adaptive expectations formulation, is an integration over time of exponentially weighted past prices; so it too is a level. Excess demand, on the other hand, is a rate, the net of consumption (purchases) per unit of time and production per unit of time. In the original model, we are told that excess demand determines $\frac{dP}{dt}$. But in a state-determined system, a rate of change (also called a decision function in system dynamics) cannot causally determine another, contemporaneous rate; only levels can determine rates. That is, the flow of excess demand (X_i) cannot be perceived except as it is accumulated (integrated) over time. For X_i to directly and causally affect the price change rate it would have to be integrated as a level, defining some "average excess demand", which does not appear in the model equations. Thus $\frac{dP}{dt}$ is a causal function only of the two system levels, price and expected price (as shown when the X_i is expanded in equation (7) and substituted in equation (5)). "Excess demand" simply defines the feedback relationships but is not modeled explicitly.

A DYNAMO flow chart of the system is shown in Figure 2, using the appropriate symbols for rates (valves), levels (rectangles) and auxiliaries (components of rates, indicated by circles).

---

[28] "A good test to determine whether a variable is a level or a rate is to consider whether or not the variable would continue to exist and to have meaning in a system that had been brought to rest." (Ibid., p. 68) In this sense, price is a level.

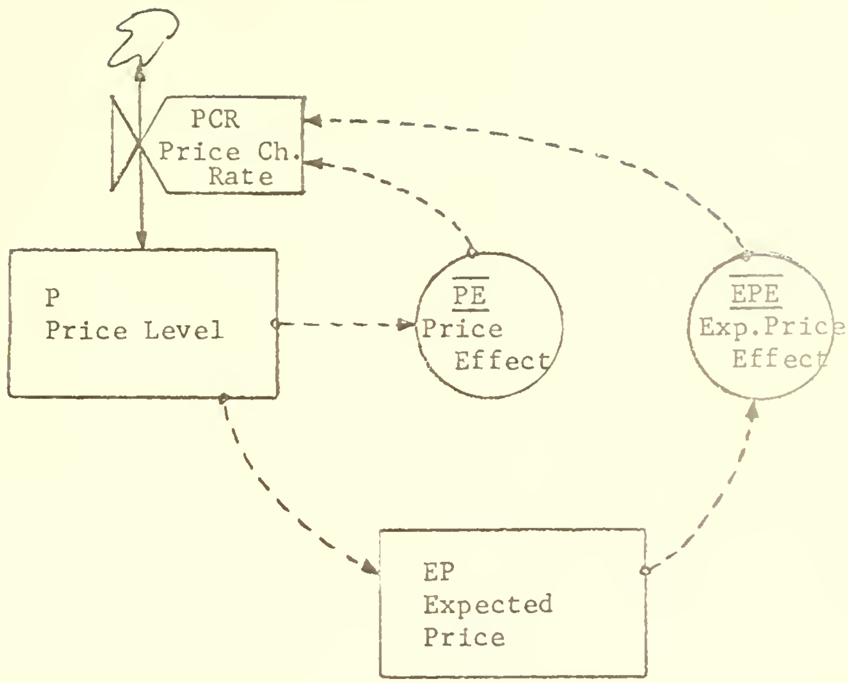
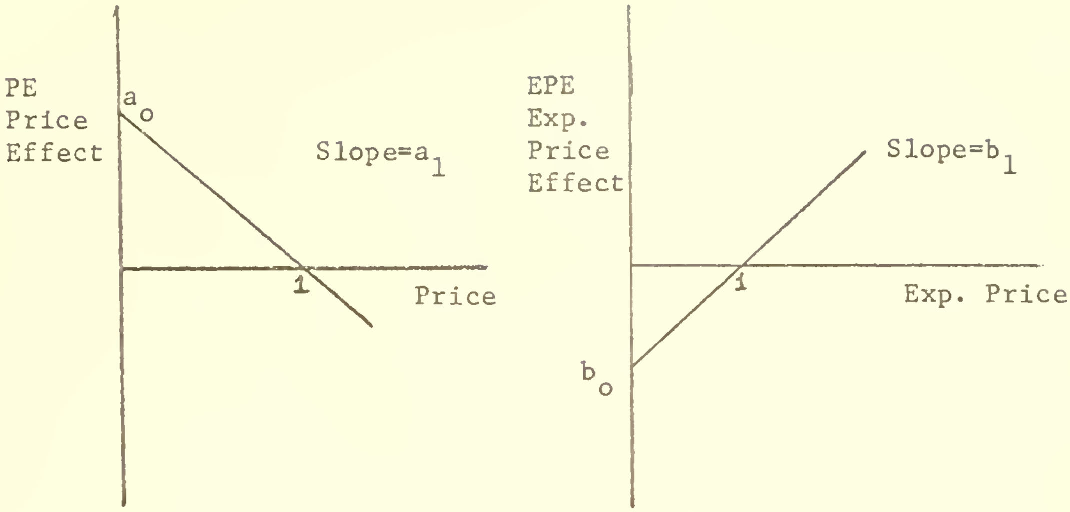


Figure 2 DYNAMO Flow Chart
of the Price-Demand Model

The price change rate PCR is defined as the sum of the two auxiliaries, PE and EPE, which are simply linear functions defined by equation (7), with slopes a_1 and b_1. They intercept the horizontal axis at the assumed normal of 1.



(a) $PE_t = a_o + a_1 P_t$ (b) $EPE_t = b_o + b_1(EP_t)$

Figure 3 Functions Relating Price and Expected Price to the Price Change Rate

The two functions are based on the feedbacks involving excess demand, as shown in Figure 1. But excess demand (a rate) does not appear explicitly in the model, since it has no explicit dynamic structure and is subsumed in the price change rate PCR.[29]

B. The Price-Inventory Model, an Extension of the Original Structure

The one-product price-demand model considered in Section A shows how one would convert the Arrow and Nerlove model into state-variable form.

---

[29] The system (which is a special case of the multi-product model) is stable only if $|a_1| > |b_1|$.

Several extensions are considered in this section in order to show how a system dynamics treatment of expectations in a macro-economic model differs from the Arrow and Nerlove approach and how the conclusions might change as a result. In the new structure, several different price expectation formulations are tested to show the importance of a causal feedback structure.

The model boundary established by Arrow and Nerlove includes market demand and the formation of price and expected price. Yet only a portion of the enclosure has a dynamic, integration-feedback structure. As shown by Samuelson, the static portion has dynamic implications pertaining to stability and damping speed; but dynamic modeling of the market would ascribe an explicit structure to the process of excess demand. One might eventually eliminate some of the structure for simplicity, but only if the eliminated structure is not important to the dynamics of the problem at hand.

To model excess demand dynamically, one must identify levels and rates in the manner described above. Net excess demand is the sum of two rates, production and consumption, which may be represented as separate flows. The integration of production minus consumption with respect to time is inventory, the accumulation of what is produced but not yet purchased. If production and consumption each responds to changing levels of price and expected price in different ways and with different timing, then inventories vary over time. Figure 4 presents the revised structure, followed by discussion of the model equations.

The price change rate can be a function only of system levels. In the previous DYNAMO version of the (single-product) Arrow and Nerlove model, price change was a function of two levels, price and expected price, reflecting

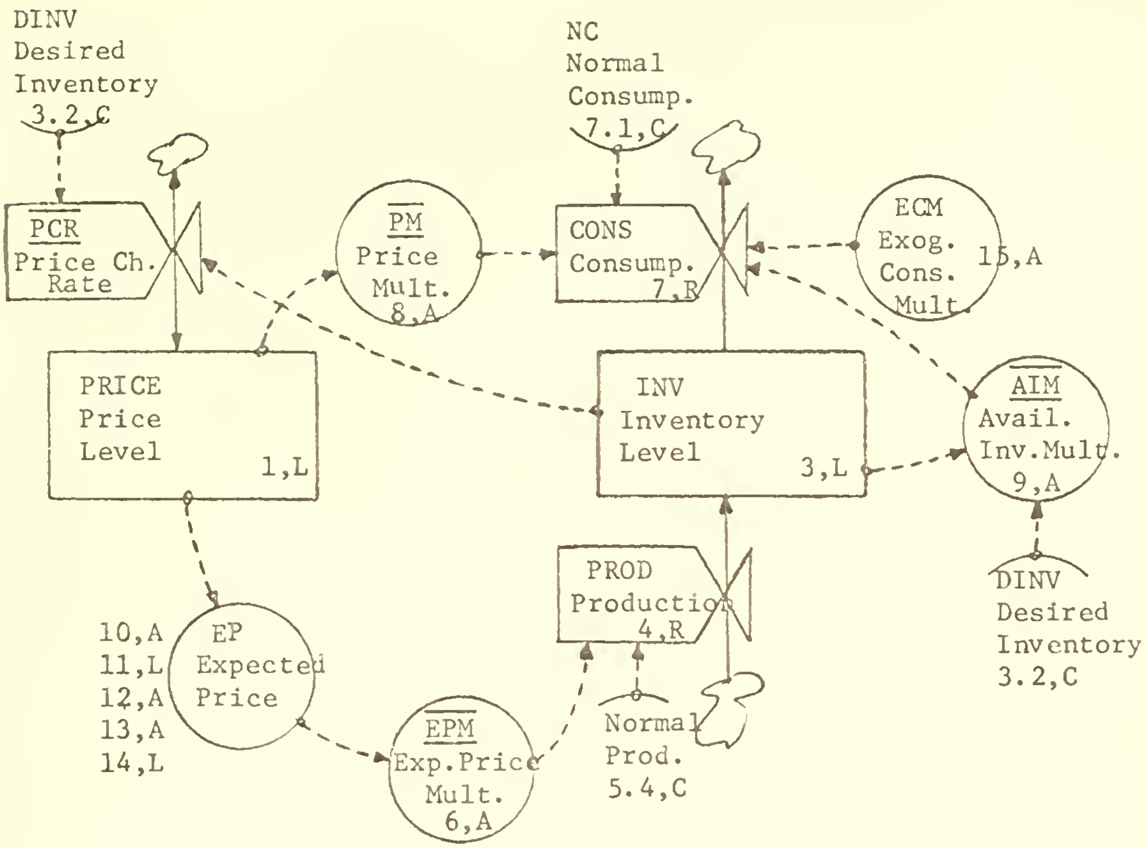


Figure 4 DYNAMO Flow-Chart of a Price-Inventory Model

a rather strange specification of causality. In the extended model, price is now determined more directly by the market demand process, that is, through the accumulation of net demand in the form of inventories.

The introduction and role of inventories in the new model is more consistent with the other modeling principles described previously. For example, the extended model more nearly represents real, observable processes. Price change is now an observed decision, representing the firm's attempt to bring inventories into line with desired levels; it is not a result of abstract,

unseen market conditions. In addition, physical flows that are endogenous to the original system are now conserved; that is, the net of supply (production) and demand (consumption) are conserved in inventories rather than ignored in static functions. The system is now explicitly dynamic and causal.

The model equations are considered below in detail.

Inventories INV are filled by production PROD and depleted by consumption CONS.

```
INV.K=INV.J+(DT)(PROD.JK-CONS.JK)                    3, L
INV=500                                              3.1, N
DINV=500                                             3.2, C
    INV     - INVENTORIES (UNITS)
    PROD    - PRODUCTION RATE (UNITS/YEAR)
    CONS    - CONSUMPTION RATE (UNITS/YEAR)
    DINV    - DESIRED INVENTORIES (UNITS)
```

PRICE is determined by the price change rate PCR, which is positive when inventories INV are below a (constant) desired level DINV and negative when inventories exceed the desired amount. Figure 5 displays the table defining the price change rate. The shape and slopes of this function reflect nothing more at this point than intuition. At the initial stage of model construction, dynamic models should emerge from a consistent, intuitive description of the processes being studied. When applying the model to a specific problem for the purpose of policy design, parameter values would be based on observation, which could include formal data analysis and consultation with people engaged in making or studying pricing decisions. The caveat stated here with respect to the PCR table also applies to the other tables described below.

```
PRICE.K=PRICE.J+(DT)(PCR.JK)                                  1, L
PRICE=1                                                       1.1, N
    PRICE  - PRICE ($/UNIT)
    PCR    - PRICE CHANGE RATE ($/UNIT-YEAR)

PCR.KL=TABHL(PCRT,INV.K/DINV,0,2,.5)                          2, R
PCRT=.3/.15/0/-.15/-.2                                        2.1, T
    PCR    - PRICE CHANGE RATE ($/UNIT-YEAR)
    INV    - INVENTORIES (UNITS)
    DINV   - DESIRED INVENTORIES (UNITS)
```

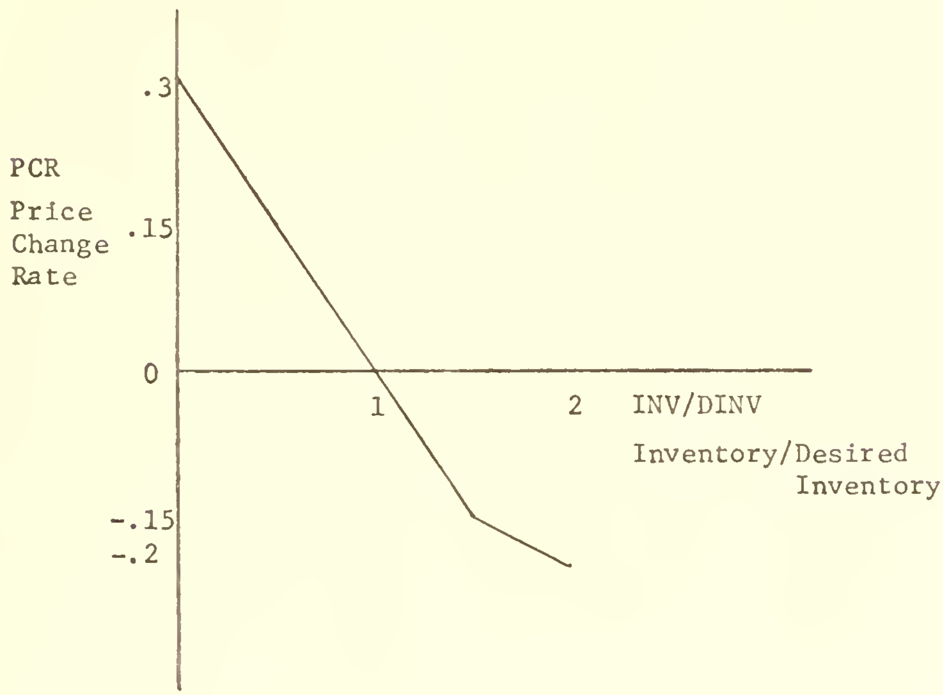


Figure 5 The Price Change Rate Table

Consumers purchase at a normal consumption rate NC, which is modulated by the available inventory multiplier AIM and the price multiplier PM. Through the available inventory multiplier AIM, consumption is reduced as inventories fall below desired levels and become less readily available. Through the price multiplier, a price

higher than the (implicit) normal price of 1($/unit) reduces consumption, while a price lower than unity encourages demand. The exogenous consumption multiplier ECM is used in the simulations to dislodge the system from equilibrium.

```
CONS.KL=NC*PM.K*ECM.K*AIM.K                                 7, R
NC=1000                                                     7.1, C
    CONS    - CONSUMPTION RATE (UNITS/YEAR)
    NC      - NORMAL CONSUMPTION RATE (UNITS/YEAR)
    PM      - PRICE MULTIPLIER (D'LESS)
    ECM     - EXOGENOUS CONSUMPTION MULTIPLIER (D'LESS)
    AIM     - AVAILABLE INVENTORY MULTIPLIER (D'LESS)
```

```
AIM.K=TABHL(AIMT,INV.K/DINV,0,1,.25)                        9, A
AIMT=0/.4/.7/.9/1                                           9.1, T
    AIM     - AVAILABLE INVENTORY MULTIPLIER (D'LESS)
    INV     - INVENTORIES (UNITS)
    DINV    - DESIRED INVENTORIES (UNITS)
```

```
PM.K=TABHL(PMT,PRICE.K,.5,1.5,.5)                           8, A
PMT=1.5/1/.5                                                8.1, T
    PM      - PRICE MULTIPLIER (D'LESS)
    PRICE   - PRICE ($/UNIT)
```

```
ECM.K=1+STEP(SH,ST)                                         15, A
SH=.2                                                       15.1, C
ST=.5                                                       15.2, C
LENGTH=0                                                    15.3, C
DT=.1                                                       15.4, C
PLTPER=.5                                                   15.5, C
    ECM     - EXOGENOUS CONSUMPTION MULTIPLIER (D'LESS)
```

The table defining the available inventory multiplier is shown in Figure 6. The function is highly nonlinear, constraining inventories to remain non-negative. It is based on the notion that as inventories become less and less available relative to normal desired levels,

consumers find it more difficult to find what they are looking for, delivery delays increase, and purchases decline below normal levels. Many similar non-linearities are found in real economic systems. However, most economic models, such as the Arrow and Nerlove example, assume linearization around some equilibrium in order to arrive at general analytical solutions. As a result, the nonlinear characteristics, like that contained in the constraints placed on consumption by depleted inventories, are lost. These non-linearities become important with excursions of the system beyond the approximated linear ranges.[30]

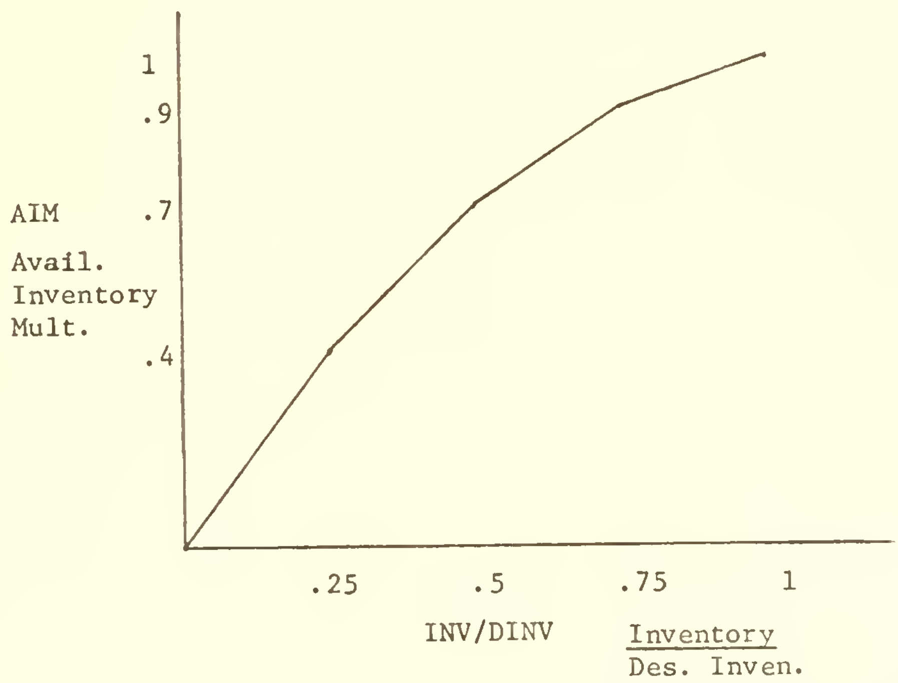


Figure 6 The Inventory Availability Multiplier Table

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[30] "When we no longer insist that we must obtain a general solution that describes in one neat package, all possible behavior characteristics of the system, the difference in difficulty between linear and non-linear systems vanishes. Simulation methods that obtain only a particular solution to each separately specified set of circumstances can deal as readily with nonlinear as with linear systems." Forrester, op. cit., p. 51.

Figure 7 displays the price multiplier table, which is considered here as a linear function over a wide range. As stated previously, the specific shape and slope would depend on what product is being modeled.

Figure 7 The Price Multiplier Table

Production PROD occurs at a normal production rate NP, except when modified by the expected price multiplier EPM. When expected price exceeds the implicit normal value of unity, production is stimulated; while an expectation below one inhibits production.

Note that this construction implies a different relationship between expected price and "excess demand" than is contained in the Arrow and Nerlove model. In their system, a higher expected price is associated with higher excess demand, and thus with a price increase ($b_{ij} \geq o$ for all i and j). The

rationale is that consumers, expecting prices to rise, will buy certain goods (presumably durables) now while the price is relatively low rather than wait, as they might, to buy later. In the revised, system dynamics model, the expected price link is with the production side of excess demand, and suggests that an increase in expected price elicits further production and so lowers, rather than raises, excess demand. The modified effect of expected price on "excess demand" suggests that, for most products, expectations play a stronger role on the production side than on purchases.

```
PROD.KL=NP*EPM.K
    PROD - PRODUCTION RATE (UNITS/YEAR)
    NP   - NORMAL PRODUCTION RATE (UNITS/YEAR)
    EPM  - EXPECTED PRICE MULTIPLIER (D'LESS)

EPM.K=TABHL(EPMT,EP.K,.5,1.5,.25)
EPMT=.5/.7/1/1.3/1.5
    EPM  - EXPECTED PRICE MULTIPLIER (D'LESS)
    EP   - EXPECTED PRICE ($/UNIT)
```

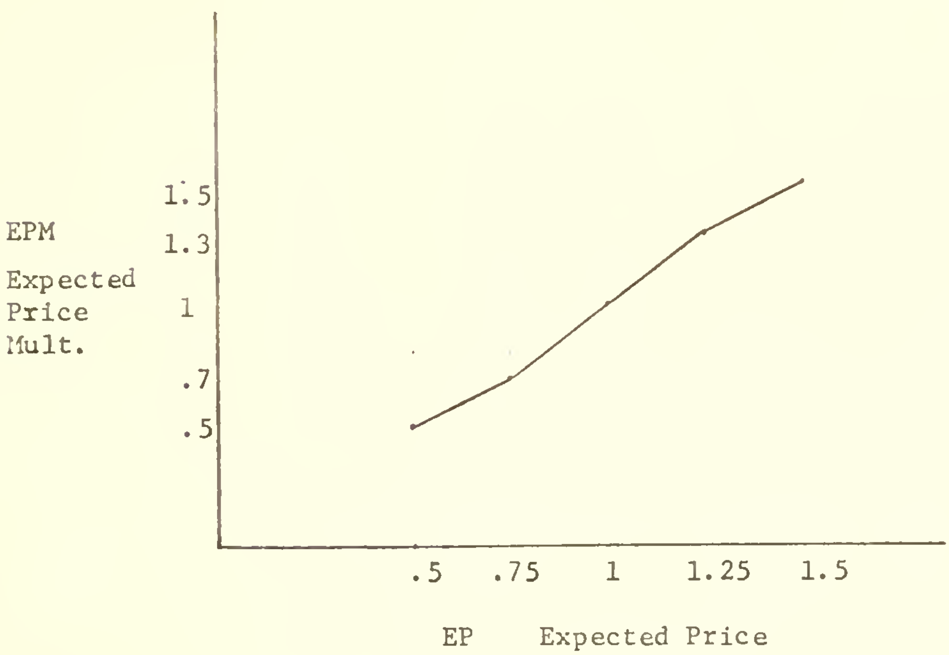


Figure 8 The Expected Price Multiplier

Expected Price EP is formed in one of five different ways. The related equations are given below and then discussed in some detail.

```
EP.K=A+B*PRICE.K+C*AP.K+D*EXP.K+E*EXAP.K                        10, A
A=1                                                              10.1, C
B=0    The parameters A through E are switches                   10.2, C
C=0    used in model simulations.                                10.3, C
D=0                                                              10.4, C
E=0                                                              10.5, C
    EP      - EXPECTED PRICE ($/UNIT)
    PRICE   - PRICE ($/UNIT)
    AP      - AVERAGE PRICE ($/UNIT)
    EXP     - EXTRAPOLATED PRICE ($/UNIT)
    EXAP    - EXTRAPOLATED AVERAGE PRICE ($/UNIT)

AP.K=AP.J+(DT/PAD)(PRICE.J-AP.J)                                 11, L
AP=1                                                             11.1, N
PAD=1                                                            11.2, C
    AP      - AVERAGE PRICE ($/UNIT)
    PAD     - PRICE AVERAGING DELAY (YEARS)
    PRICE   - PRICE ($/UNIT)

EXP.K=PRICE.K+(PET/PAD)(PRICE.K-AP.K)                            12, A
PET=.5                                                           12.1, C
    EXP     - EXTRAPOLATED PRICE ($/UNIT)
    PRICE   - PRICE ($/UNIT)
    PET     - PRICE EXTRAPOLATION TIME (YEARS)
    PAD     - PRICE AVERAGING DELAY (YEARS)
    AP      - AVERAGE PRICE ($/UNIT)

EXAP.K=AP.K+(APET/SPAD)(AP.K-SAP.K)                              13, A
    EXAP    - EXTRAPOLATED AVERAGE PRICE ($/UNIT)
    AP      - AVERAGE PRICE ($/UNIT)
    APET    - AVERAGE PRICE EXPECTATION TIME (YEARS)
    SPAD    - SMOOTHED AVERAGE PRICE DELAY (YEARS)
    SAP     - SMOOTHED AVERAGE PRICE ($/UNIT)

SAP.K=SAP.J+(DT/SPAD)(AP.J-SAP.J)                                14, L
SAP=1                                                            14.1, N
SPAD=1                                                           14.2, C
APET=.5                                                          14.3, C
    SAP     - SMOOTHED AVERAGE PRICE ($/UNIT)
    SPAD    - SMOOTHED AVERAGE PRICE DELAY (YEARS)
    AP      - AVERAGE PRICE ($/UNIT)
    APET    - AVERAGE PRICE EXPECTATION TIME (YEARS)
```

The alternative formulations for expected price EP are:[31]

(a) EP = 1

(b) EP = PRICE

(c) EP = AP (AP - exponentially averaged price)

(d) EP = EXP (EXP - extrapolated price)

(e) EP = EXAP (EXAP - extrapolated average price).

In a specific modeling situation, one's choice among the five alternatives would depend on how one thinks people perceive prices and trends, how long it takes price changes to be noticed, and the weight people place on current and past observations in forming their expectations. The first three formulations are forms of adaptive expectations. For example, if we set $\epsilon = 0$ in the solution for EP,

$$EP(t) = EP(0)e^{-\epsilon t} + e^{-\epsilon t}\int_0^t \epsilon P(u)e^{\epsilon u}du, \qquad (4),$$

then EP = EP(0) = 1.

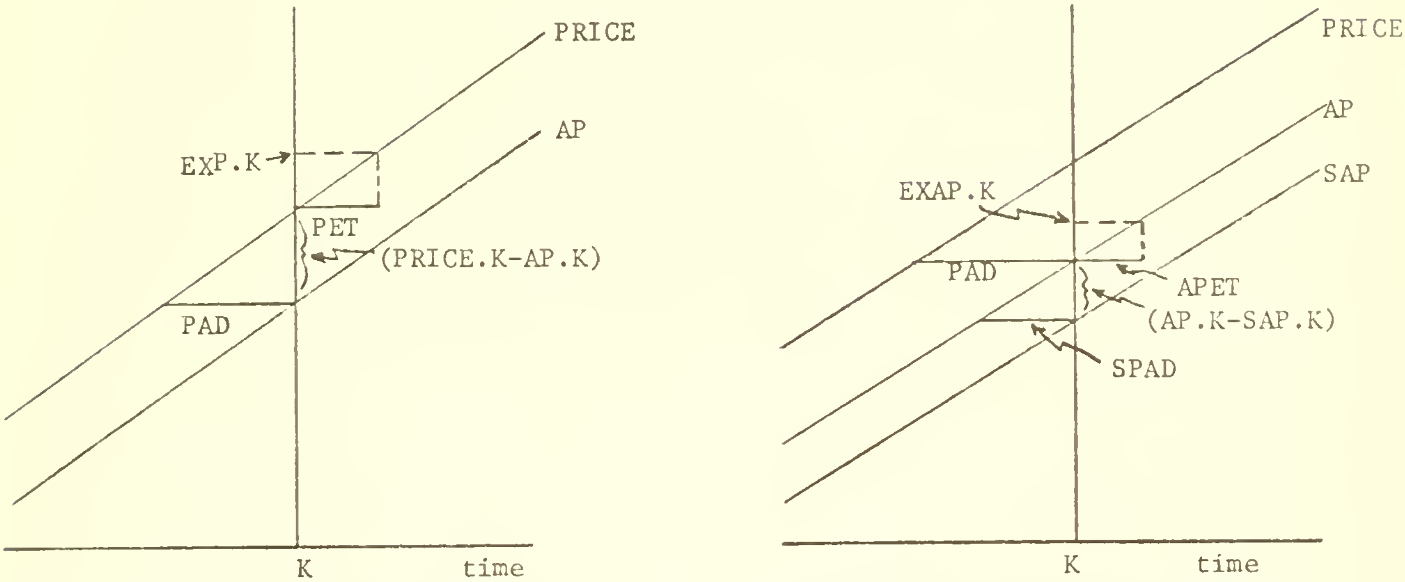
Setting $\epsilon = \infty$ is equivalent to a response time of zero in the differential equation for EP; and EP = PRICE. Alternative (c) constitutes an exponentially averaged price AP, which can be related by the response time (the price averaging delay PAD) to the movement of price over time. Thus if price oscillates, average price will also oscillate, with the same frequency but with a phase shift and change in amplitude determined by the delay/frequency ratio.[32]

---

[31] Two of these formulations have exact counterparts in the economics literature. The second (EP = PRICE) corresponds to the "classical" expectations format, where $EP_t = P_{t-1}$; the third is Nerlove's adaptive expectations, represented discretely as $EP_t = \alpha \sum_{j=1}^{\infty}(1-\alpha)^{j-1}P_{t-j}$ and shown in its continuous form in equations (4).
See Muth, _op. cit._, p. 332.

[32] First-order delays are treated more fully in Forrester, _op. cit._, pp. 86-92; and phase and gain relationships are described on pp. 415-17.

Expected price formulations (d) and (e) are both extrapolations of past trends and are described graphically in Figure 9:



(a) $EXP.K = PRICE.K+PET*\left(\frac{PRICE.K-AP.K}{PAD}\right)$

(b) $EXAP.K = AP.K+APET*\left(\frac{AP.K-SAP.K}{SPAD}\right)$

KEY:

EXP - Extrapolated Price
AP - Average Price
PAD - Price Averaging Delay
PET - Price Extrapolation Time

EXAP - Extrapolated Avg. Price
AP - Average Price
PAD - Price Averaging Delay
SAP - Smoothed Average Price
SPAD - Smoothed Price Avg. Delay
APET - Avg. Price Extrap. Time

Figure 9. Extrapolative Forecasting[33]

In Figure 9(a), current price rises at a constant rate over time (for ease of exposition) and is extrapolated into the future at some past rate of change. That is, price is compared with average price AP and is extended

---

[33] A similar drawing describing extrapolative forecasting appears in Ibid., p. 439. Since AP and SAP are exponential averages, the two extrapolative forecasts shown here are not equivalent to the extrapolative expectations of economic models, one version of which is $EP_t=(1-\alpha)\ P_{t-1}+\alpha P_{t-2}$ $(-1<\alpha<1)$;

into the future at the slope $(\frac{PRICE.K - AP.K}{PAD})$. In this case extrapolated price EXP at time K is always higher than price when the price expectation time PET > 0, the exact position depending on PET and PAD.

In Figure 9(b), average price AP, rather than price, is extrapolated by the observed past rate of change. Here the expected price (extrapolated average price EXAP) at time K will fall above average price when the average price extrapolation time APET > 0. Expected price will exceed current price when APET > PAD.

The causal loop diagram in Figure 10 shows that price is embedded in two negative feedback loops. If each of these loops is considered separately, an increase in price creates self-limiting or negative feedback pressures on itself. Thus a higher price reduces consumption, which increases inventories, which causes price to decline. Coupling the three negative loops can generate varying behavior modes, as shown in the system simulations.

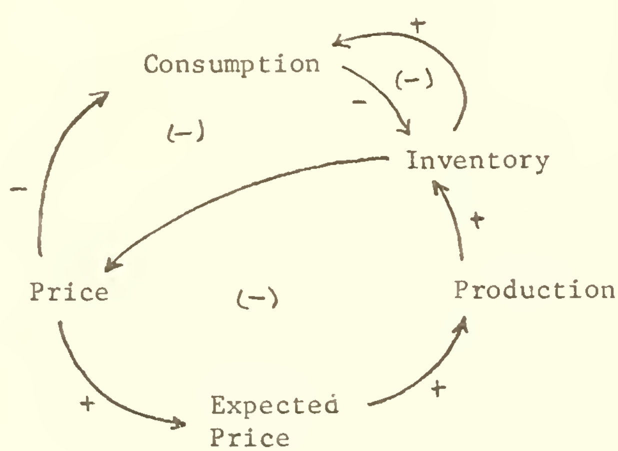


Figure 10. Causal Loop Diagram of a Price-Inventory Model

The model described above was simulated over time to show the effects of the five expected price formulations (including exponential averaging with two different time constants). The initial equilibrium[34] was disturbed by a 20% step in consumption CONS (i.e. by stepping the exogenous consumption multiplier ECM from 1 to 1.2). The simulated time paths of inventories are shown in Figure 11.

In all cases, inventories fluctuate about an equilibrium value of 500. But depending on the expectations model, the oscillations are of constant, slightly increasing, or decreasing amplitude. Thus the system is convergent, or asymptotically stable, when expected price equals

(a) a constant (=1)

(b) current price

(c) average price with a delay of 6 years (i.e. with adaptive expectations "elasticity" of 1/6)

(d) extrapolated current price.

The system is unstable, with slightly divergent oscillations when price equals

(e) extrapolated average price.

The model displays steady-state oscillations when price equals

(f) average price with a delay of two years ("elasticity" of 1/2).

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[34] Where PRICE=EP=1, CONS=PROD=1000, and INV=DINV=500

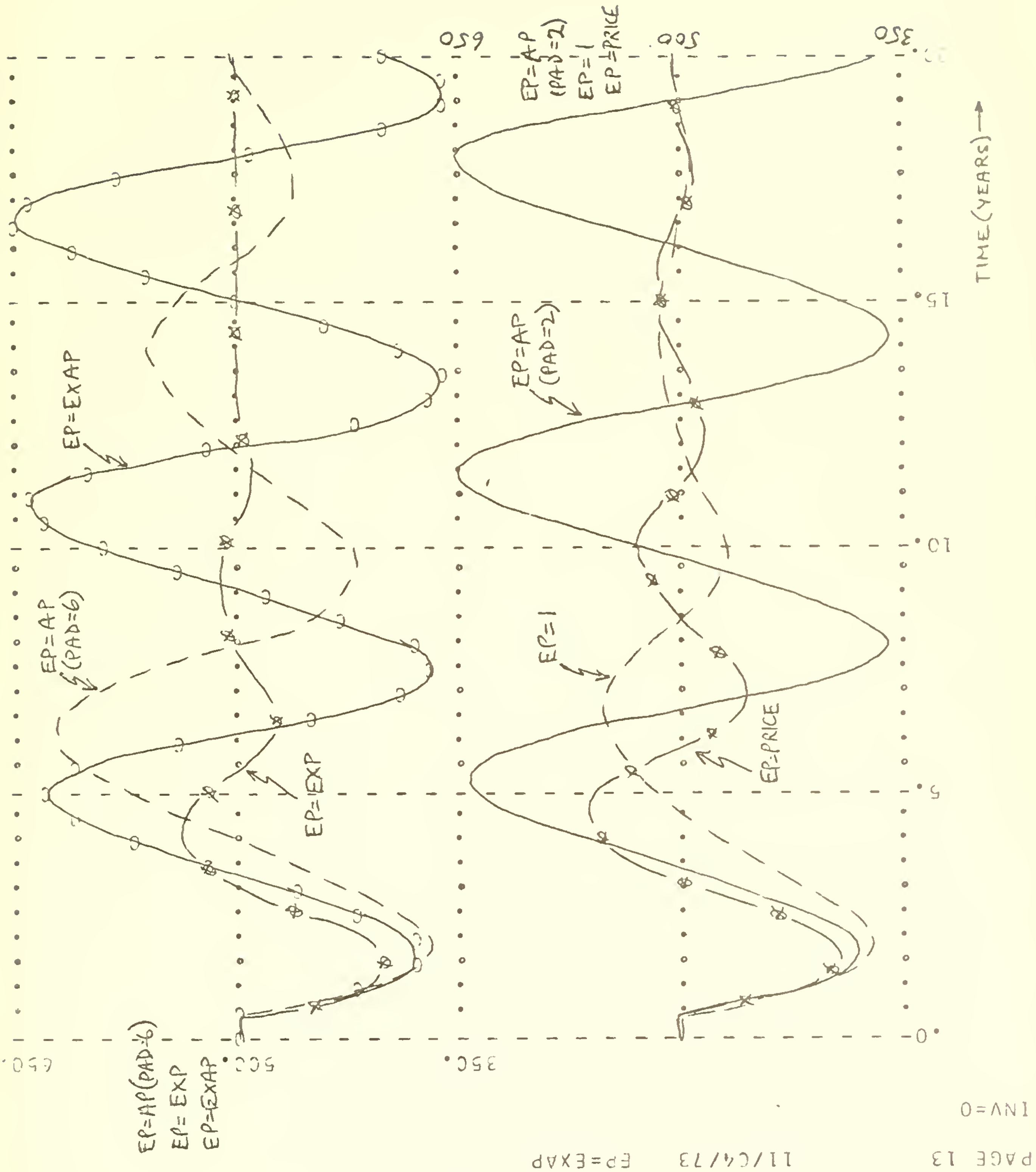


Figure 11. Inventory Time-Paths with Different Expectation Formulations

In this model, the stability effects of the adaptive expectations formulations (c) and (f) depend on the price averaging delay PAD. This can be demonstrated by examining Figures 12 and 13. In Figure 12, the five plotted variables oscillate in an undamped fashion. Production PROD moves in phase with expected price EP, because of the slope of the expected price multiplier table and because production depends only on expected price. Consumption CONS moves 180° out of phase with PRICE, because of the direct negative causality. Because expected price EP is a first-order smoothing of PRICE, and the ratio of the delay to the frequency is about .3, the turning points of price lead those of the expected level by roughly 60°. Thus production and consumption are approximately 240° out of phase, and inventories oscillate accordingly.

In Figure 13, the price averaging delay PAD equals 6 rather than 2. The higher delay/frequency ratio means that expected price now lags price by around 80°, so that the effects of early price changes on production (through EP) take longer to be realized. The longer delay also affects production because of its impact on the amplitude of expected price relative to the amplitude of price.[35]
Expected price does not peak as high relative to price when the delay is six years as it does when the delay is two years. Hence, production, which depends on expected price peaks at a lower point, inventories do not get as high as in the previous case, and price does not fall as low. With price

[35] If the price input into the price averaging filter were sinusoidal, with a frequency of 6 years, the amplitude ratio would be about .2 with the six year delay, vs. about .5 with the two year delay.

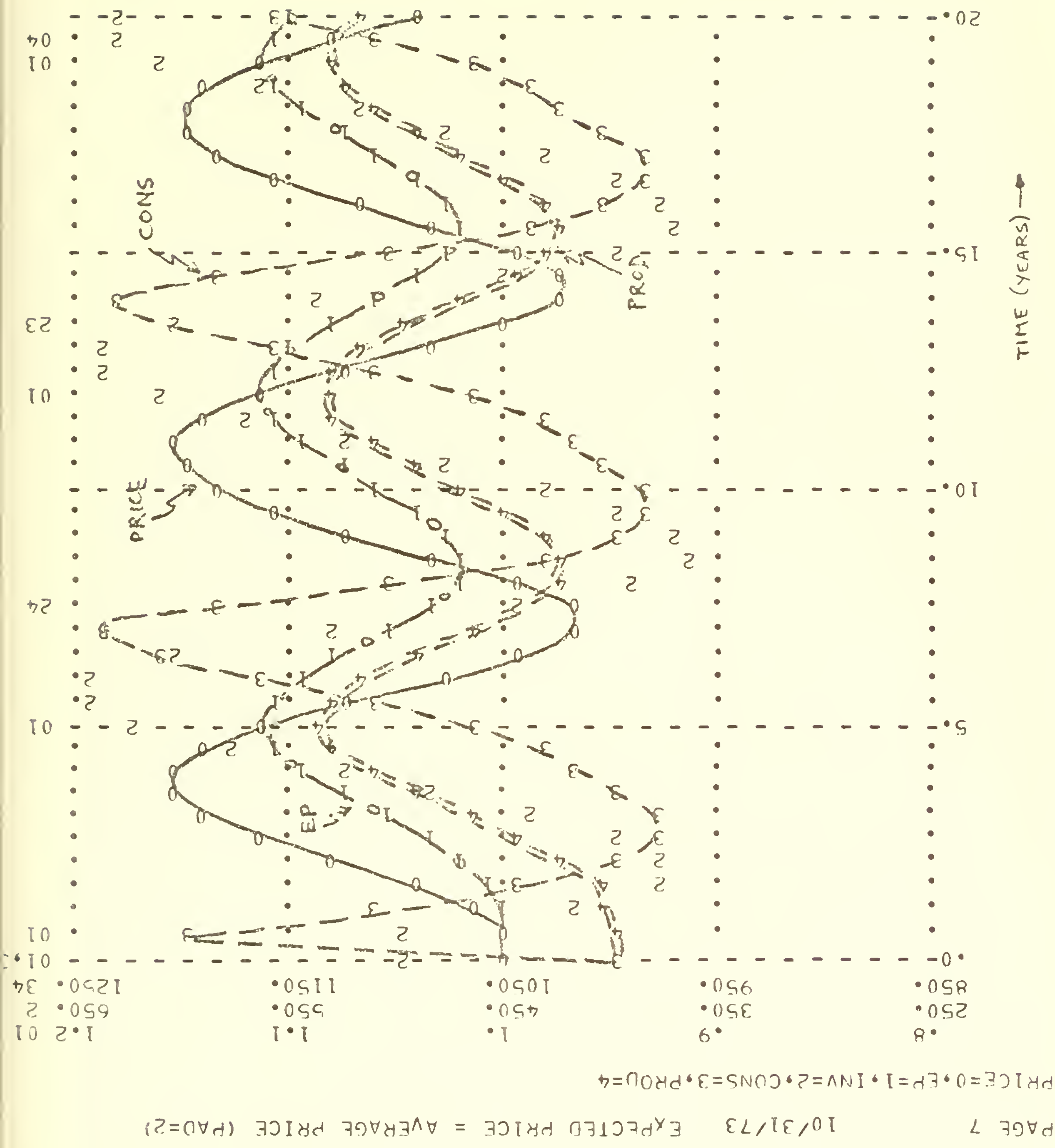


Figure 12. Simulation of Price-Inventory Model Using Adaptive Expectations (with Delay=2 Years)

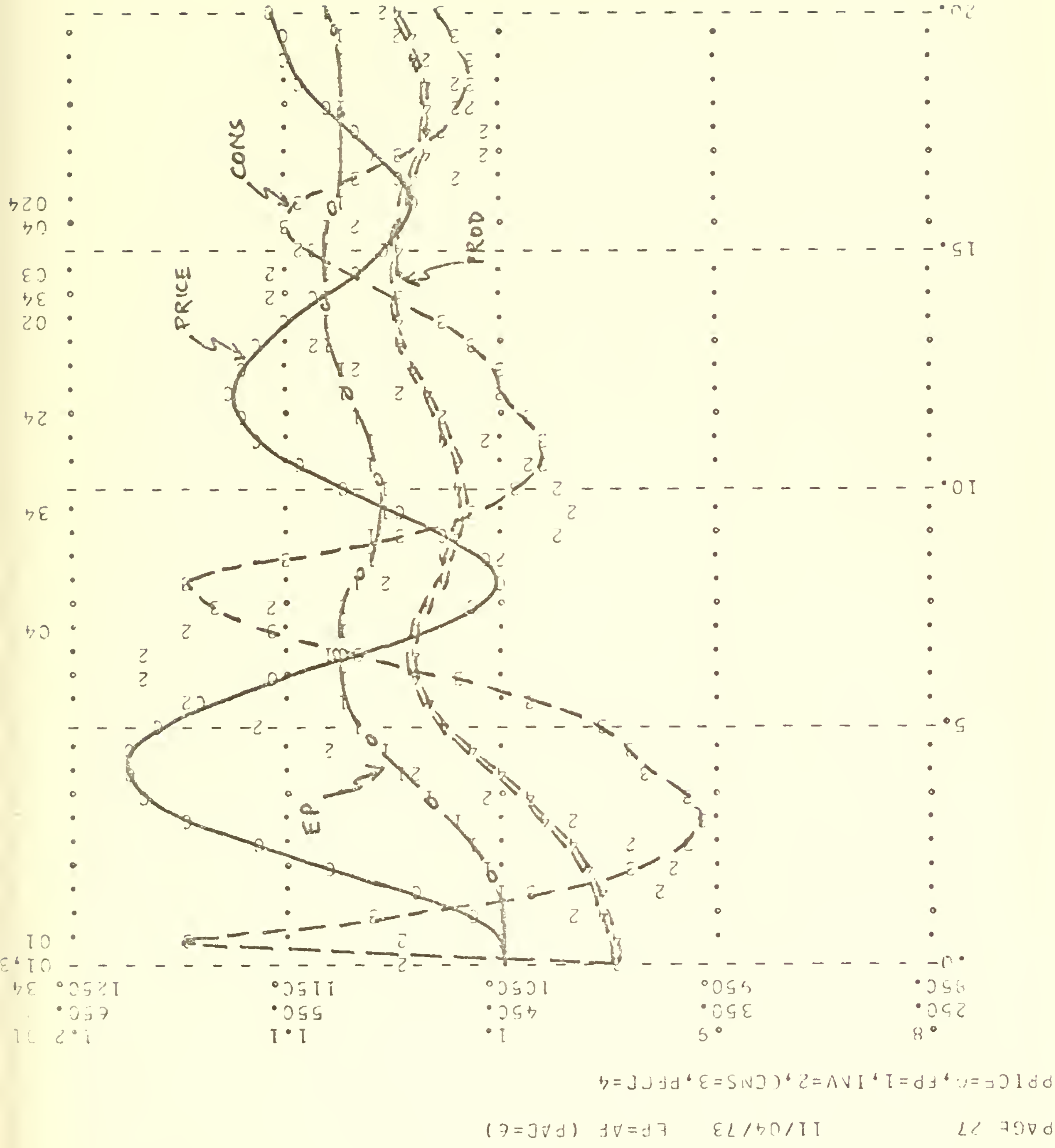


Figure 13. Simulation of Price-Inventory Model Using Adaptive Expectations (with Delay=6 Years)

declining less sharply, consumption is not as strongly stimulated, inventories decline by less, and subsequent price movements converge toward a new equilibrium (about 7% above the initial value). Fluctuations in consumption and production are damped so that the two rates approach their new equilibrium of about 1100 (units/year).

The above analysis indicates the kinds of issues one must consider in dealing with the effects of different expected price formulations in an integration-feedback model. Without going into similar detail for other cases, it should be noted that the extrapolation of current price (case (d)) yields the greatest system stability, even though it might appear in isolation to be the least stable formulation.[36] System stability results from the quicker response of production to price change (through the expected price multiplier EPM), which causes production and consumption to remain almost 180° out of phase (rather than 240° or 260° as in the price averaging cases). The swings in inventories are less intense and quickly converge toward equilibrium.

C. Modifications of the Price-Inventory Model

This section will show that simple changes in the Price-Inventory Model can substantially alter the effects of different expectation formulations on system behavior. One modification is to make production dependent on relative price. The relative nature of prices as they affect market decisions is always implicit in economic models, but seldom explicit.

---

[36] "Stable" in this context describes "the pattern of revisions of future forecasts in response to current surprises," not the effects of expectation formulations on the total system. [Mincer, J. (Ed.). Economic Forecasts and Expectations. (New York: Columbia University Press, 1969), p. 90.] Extrapolation of the current price at the recently observed change in price is relatively unstable in Mincer's sense, because any change in price immediately changes expectations by the same amount or more. Adaptive expectations is more stable as it involves revisions by the same amount or less.

For example, if higher prices are said to increase supply or reduce demand, we must ask, "higher than what?" The conventional equilibrium model relates price to the equilibrium value, although it is doubtful that consumers or producers compare current prices with some system-determined equilibrium.

The system dynamics model is likely to reference current price to an "average", "normal", or "expected" price based on past observation. The "normal" might be approximated by a constant. This is the case in the previous expected price multiplier table, which is normalized around 1. But its meaning is different from a system equilibrium which is not drawn in an operating sense on past observation. The reference in this extension is no longer an implied constant value, but the expected value, as described in the previous section. Production now depends on price relative to expected price, rather than on expected price (relative to an implied "normal" of 1). Price now becomes an element in three, rather than two, negative feedback loops:

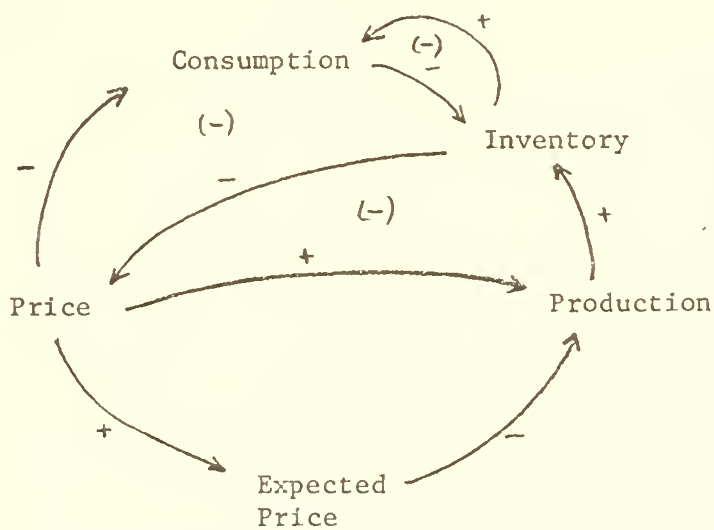


Figure 14. Causal Loop Diagram of the Modified Price-Inventory Model

The polarities of the price-production and expected price-production links reveal a more complex decision pattern among producers. Firms now expand their output if they expect future prices to be lower than current prices, in order to benefit from the relatively good conditions prevailing today. On the other hand, if the expected price exceeds today's level, current output is curtailed in favor of future output. The new production rate is a function of relative prices, as defined by the relative price multiplier RPM.

```
PROD.KL=NP*RPM.K

        PROD   - PRODUCTION RATE (UNITS/YEAR)
        NP     - NORMAL PRODUCTION RATE (UNITS/YEAR)
        RPM    - RELATIVE PRICE MULTIPLIER (D'LESS)

RPM.K=TABHL (RPMT,EP.K/PRICE.K,.8,1.2,.2)

RPMT=1.4/1/.6

        EP     - EXPECTED PRICE ($/UNIT)
        PRICE  - PRICE ($/UNIT)
```

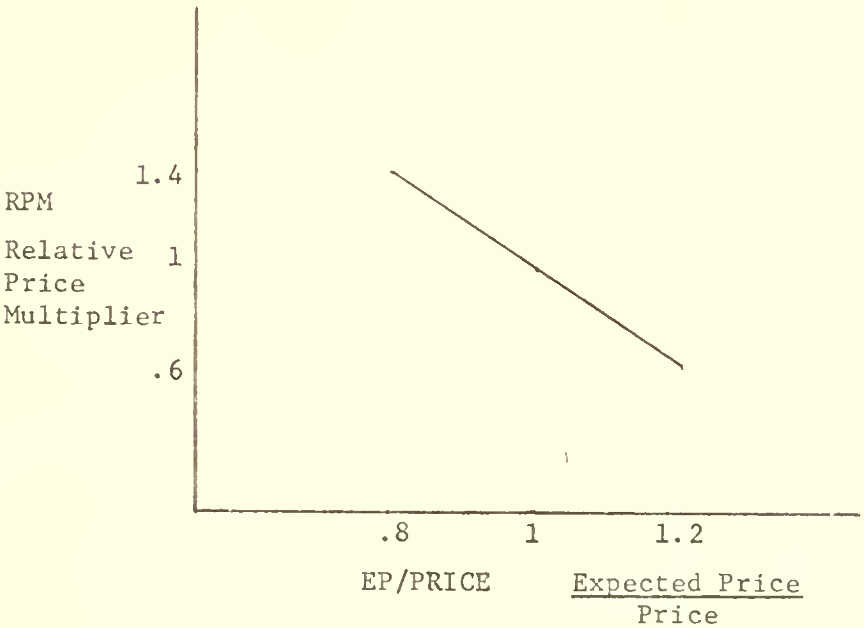


Figure 15. The Relative Price Multiplier Table

The floating reference implicit in the relative price multiplier RPM illustrates that there is no single-valued function relating current prices to production rates. This reflects an instance of hysteresis.[37] When expected price, for example, equals an exponential average of past prices, EP lags behind PRICE, as shown in Figure 16.

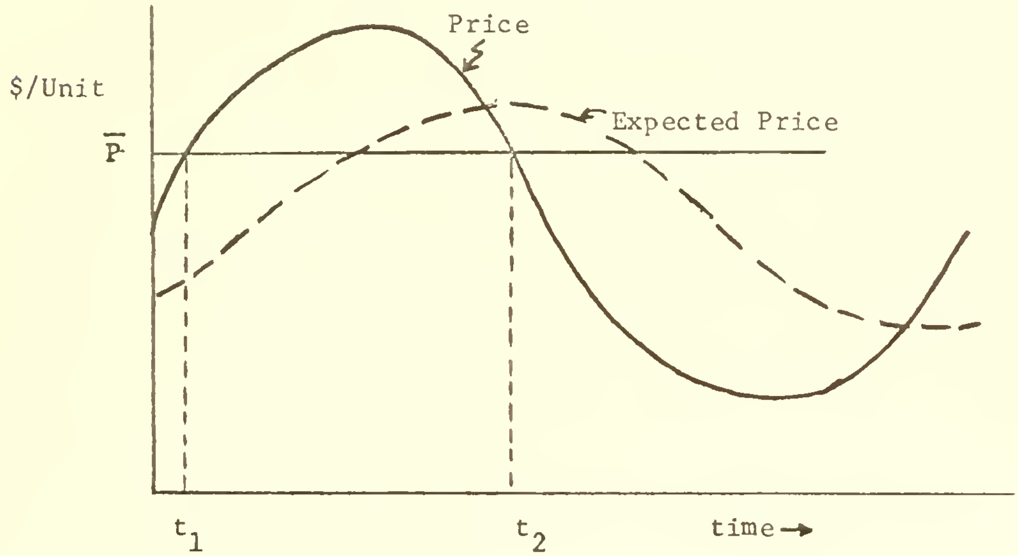


Figure 16. An Illustration of Hysteresis

A horizontal line drawn from $\bar{P}$ crosses the price curve at times t_1 and t_2. At t_1, current price is rising and exceeds the expected level; the effect is expanded production. At t_2, price is falling and is below the expected value; the effect is reduced output. In other words, as the

---

[37] This illustration draws on a similar example contained in a draft paper by Mass, N.J., entitled "Short-term and Long-term Influence of Money Wage and Real Wage on Voluntary Work Offers," (System Dynamics Group, M.I.T., January 1973), pp. 6-7.

current price retraces its former values through time, the influence of price on production does not also retrace its former path.

Figure 17 reveals the behavior of inventories in the modified structure, using the same six expectations models as in the first simulations. Now all of the runs are rapidly damped, except for the extrapolated price EXP formulation, which exhibits steady-state oscillations. Stability characteristics of the system are basically different for three of the six simulations:

(1) Extrapolated average price EXAP had the greatest destabilizing effect in the first version of the price-inventory model, but now causes the system to equilibrate most rapidly.

(2) Average price AP, with PAD=2, exhibited steady-state oscillations in the first version but now equilibrates rapidly.

(3) Extrapolated price EXP, which had the greatest stabilizing influence in the first case, is now the only structure that prevents the system from attaining equilibrium.

A second modification is now introduced. Suppose production, like consumption, is influenced by the current level of inventories INV relative to desired inventories DINV (i.e. producers no longer determine output simply on the basis of relative prices). This feedback from inventories to production is expressed through the inventory multiplier IM, which reduces production when INV>DINV and stimulates production when INV<DINV.

```
PROD.KL=NP*RPM.K*IM.K                                      4, R
    PROD   - PRODUCTION RATE (UNITS/YEAR)
    NP     - NORMAL PRODUCTION RATE (UNITS/YEAR)
    RPM    - RELATIVE PRICE MULTIPLIER (D'LESS)
    IM     - INVENTORY MULTIPLIER (D'LESS)

IM.K=TABHL(IMT,INV.K/DINV,.5,1.5,.5)                       5, A
IMT=1.5/1/.5                                               5.1, T

NP=1000                                                    5.4, C
    IM     - INVENTORY MULTIPLIER (D'LESS)
    INV    - INVENTORIES (UNITS)
    DINV   - DESIRED INVENTORIES (UNITS)
    NP     - NORMAL PRODUCTION RATE (UNITS/YEAR)
```

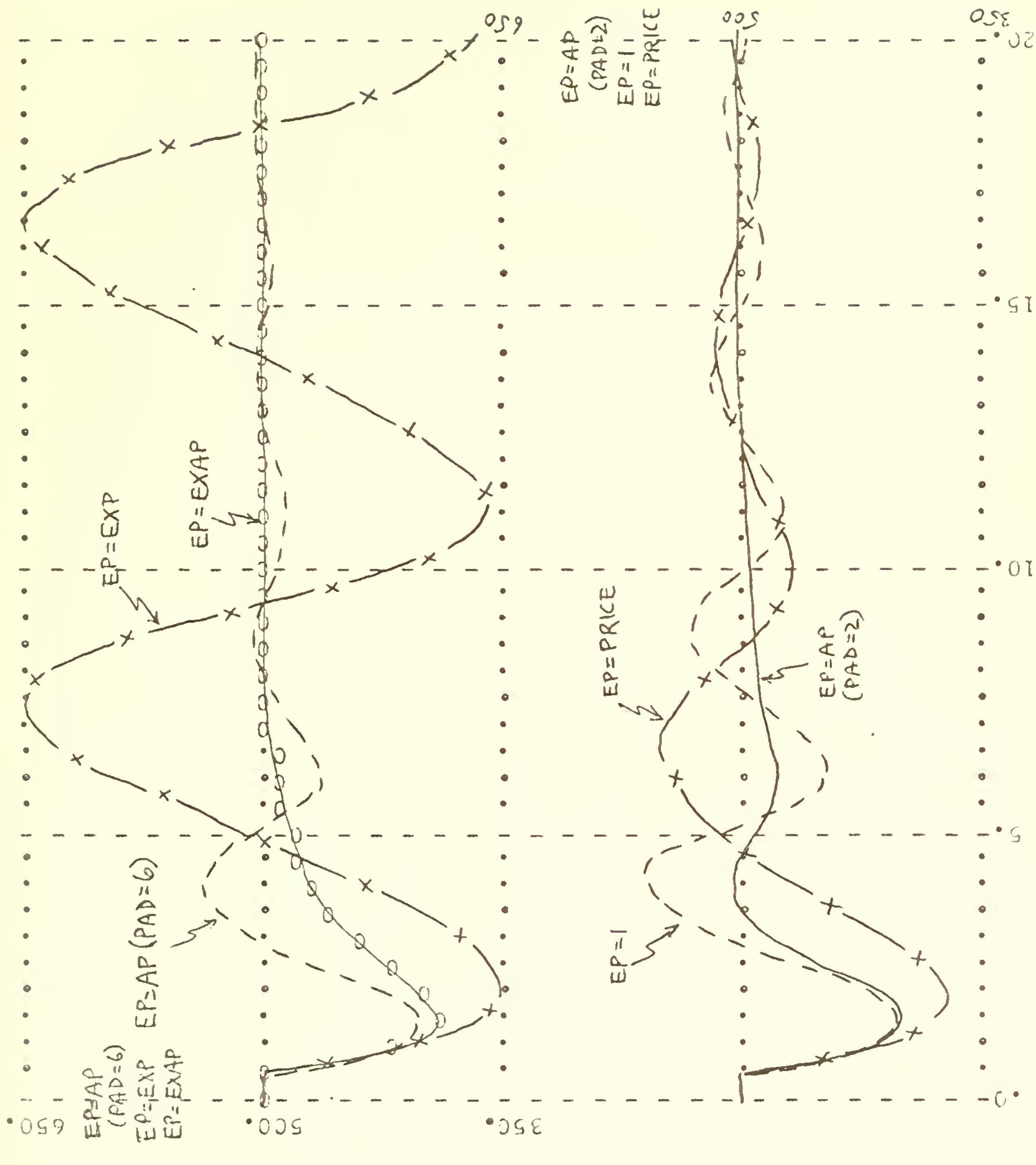


Figure 17. Inventory Time-Paths with Different Expectation Formulations
(Modified Model)

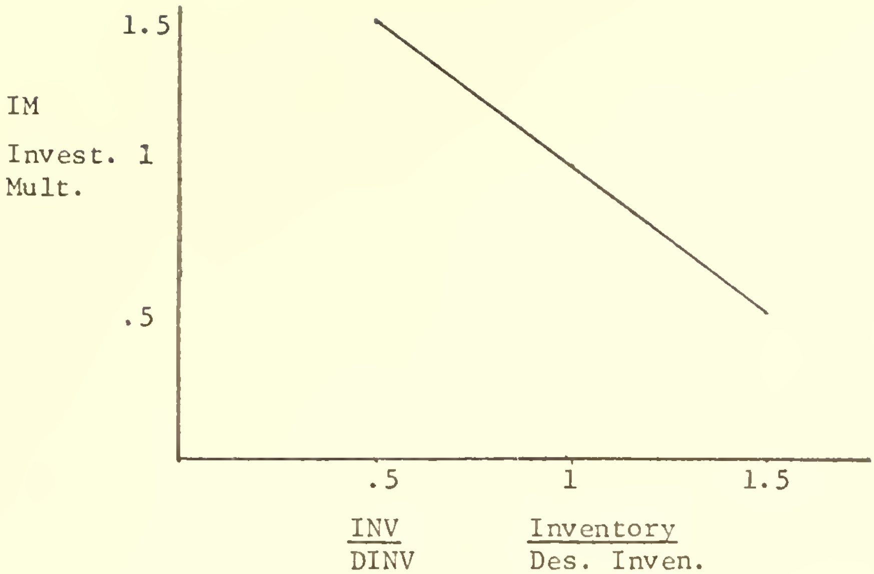


Figure 18. The Inventory Multiplier Table

The DYNAMO flow chart of Figure 4 is now modified as follows:

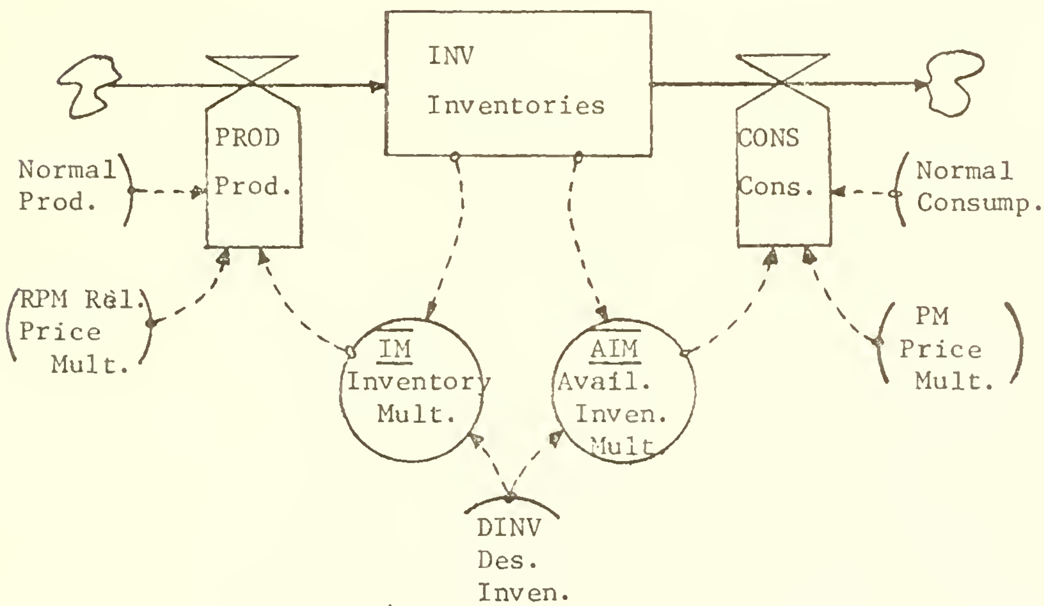
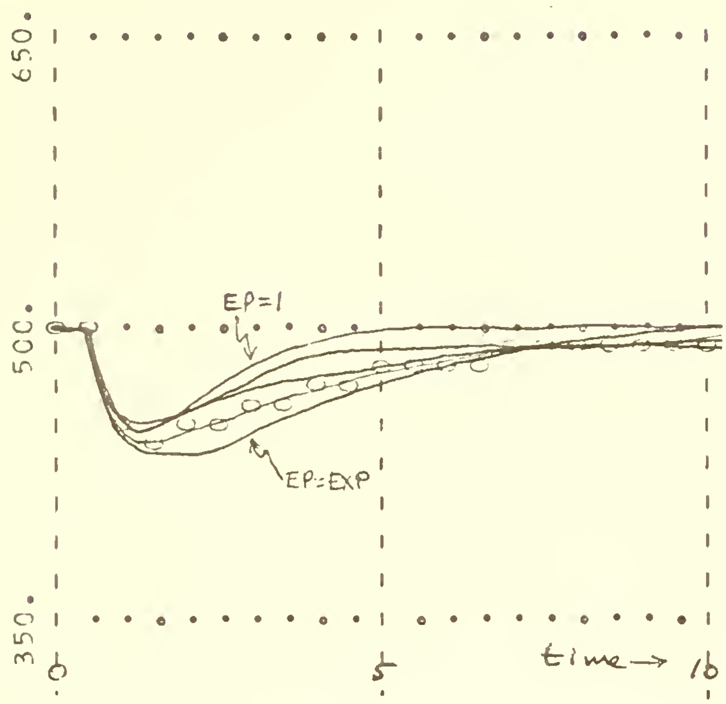


Figure 19. A Second Modification of the Production Decision

The addition of an inventory-production feedback now tightly damps <u>all</u> of the six simulations, as revealed in Figure 20:



<u>Figure 20.</u> <u>Nearly Identical Inventory Time-Paths for Modified System Structure</u>

If one were confident about each particular formulation in this final version, and if the behavior of the important variables resembled that observed in the real world, then, for this system, the way in which price expectations are formed would be relatively insignificant. One would not be justified in doing extensive research on expectations relative to the market in question, but would do better to look at other parts of the model for an understanding of why the system behaves as it does.

GWL:mer